Small-Angle Scattering [mln105]

Scattering angle from transverse momentum: $\sin\theta = \frac{p_y}{p} \quad \Rightarrow \; \theta = \frac{p_y}{mv_0} + \ldots$

Impact parameter: s.

Impulse and transverse momentum: $p_y = \int_{-\infty}^{+\infty} dt\, F_y$.

Transverse force: $F_y = -\frac{\partial V}{\partial y} = -\frac{dV}{dr}\frac{\partial r}{\partial y} = -\frac{dV}{dr}\frac{y}{r}, \quad r = \sqrt{x^2+y^2+z^2}.$

Amount of transverse motion during collision assumed negligible: $F_y = -\frac{dV}{dr}\frac{s}{r}$.

Change in speed of particle during collision assumed negligible:

$$dt = \frac{dx}{v_0} \quad \Rightarrow \; p_y = -\frac{s}{v_0}\int_{-\infty}^{+\infty}\frac{dV}{dr}\frac{dx}{r}.$$

Eliminate dx: $x = \sqrt{r^2 - s^2} \quad \Rightarrow \; \frac{dx}{dr} = \frac{r}{\sqrt{r^2-s^2}}$.

Transverse momentum: $p_y = -\frac{2s}{v_0}\int_s^{+\infty}\frac{dV}{dr}\frac{dr}{\sqrt{r^2-s^2}}$.

Scattering angle: $\theta(s) = -\frac{s}{E}\int_s^{+\infty}\frac{dV}{dr}\frac{dr}{\sqrt{r^2-s^2}}, \qquad E = \frac{1}{2}mv_0^2$.

Scattering cross section: $\sigma(\theta) = \frac{s(\theta)}{\theta}\left|\frac{ds}{d\theta}\right|$ with $s(\theta)$ from inversion of $\theta(s)$.

Application to power-law potential: [mex246]