Velocity Vector Field in Phase Plane

Phase portraits are readily produced computationally. We use the StreamPlot command of Mathematica for a demonstration. Consider a particle of unit mass moving in a quartic potential,

\[ V(x) = ax - bx^2 + cx^4. \]

For the parameter values indicated, we have an asymmetric double-well potential as shown. The presence of three fixed points is readily recognized. Zooming into their vicinity reveals their nature.

The components of the velocity vector field are \( \dot{x} = y \) and \( m \dot{y} = -dV/dx \). The presence of three fixed points is readily recognized.
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Adding damping changes the force to $F = -dV/dx - \gamma \dot{x}$. We use $\gamma = 0.5$. The hyperbolic fixed point is still present. The two elliptic fixed point turn into spiral attractors.