Dynamical Systems with 1 Degree of Freedom [mln14]

Newton's equation of motion: $m\ddot{x} = F(x, \dot{x}) \;\Leftrightarrow\; \dot{x} = y,\; \dot{y} = F(x, y)/m$.

Velocity vector field in 2D phase space: $(\dot{x}, \dot{y})$.

Solution $(x(t), y(t))$ describes *trajectory* in 2D phase space.
All trajectories are tangential to velocity vector field.
Trajectories do not intersect each other or themselves.

Orbits are projections of trajectories onto the x-axis.

Fixed points in phase space have zero phase velocity: $(\dot{x}, \dot{y}) = (0, 0)$.

Conservative system: $F = F(x) = -\dfrac{dV}{dx}, \quad V(x) = -\displaystyle\int_{x_0}^{x} dx\, F(x)$.

Integral of the motion: $E(x, \dot{x}) = \dfrac{1}{2}m\dot{x}^2 + V(x) = \text{const.}$

In conservative systems, trajectories are confined to lines of constant energy.

Separatrix: line of constant energy corresponding to local maximum of $V(x)$.

In conservative systems, there are two types of fixed points:
elliptic fixed points at energies where $V(x)$ has a local minimum.
hyperbolic fixed points at energies where $V(x)$ has a local maximum.

In dissipative systems, there are additional types of fixed points:
attractors and *repellors*.

Not all attractors are fixed points:
Spirals, stars, and *nodes* are 0D attractors in 2D phase space.
Limit cycles are 1D attractors in 2D phase space.