Variational Problem with Auxiliary Condition [mln16]

Search for a function $y(x)$ that yields an extremum of the integral

$$J = \int_{x_1}^{x_2} dx\, f[y(x), y'(x); x]$$

subject to an auxiliary condition in the form of the integral constraint

$$C = \int_{x_1}^{x_2} dx\, \sigma[y(x), y'(x); x] = \text{const.}$$

Use the functional $F_\lambda[y(x), y'(x); x] = f[y(x), y'(x); x] + \lambda\sigma[y(x), y'(x); x]$, where λ is an undetermined Lagrange multiplier.

Find the extremum of $J_\lambda = \displaystyle\int_{x_1}^{x_2} dx\, F_\lambda[y(x), y'(x); x]$.

This leads to Euler's equation $\displaystyle\frac{\partial F_\lambda}{\partial y} - \frac{d}{dx}\left(\frac{\partial F_\lambda}{\partial y'}\right) = 0.$

Then adjust the value of λ in the solution such that the auxiliary condition is satisfied.

Examples:

- Isoperimetric problem [mex28]
- Catenary problem [mex38]