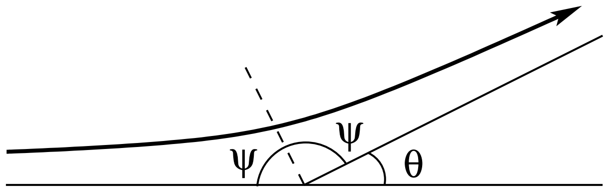
Determination of scattering angle [mln20]

Orbital integral: $\vartheta = \int_\infty^r dr \frac{\ell/mr^2}{\sqrt{\frac{2}{m}\left[E - V(r) - \frac{\ell^2}{2mr^2}\right]}}.$

Periapsis: $\vartheta(r_{min}) \doteq \psi \quad \Rightarrow \; 2\psi + \theta = \pi.$



$$\Rightarrow \;\psi = \int_{r_{min}}^\infty \frac{dr/r^2}{\sqrt{\frac{2m}{\ell^2}[E - V(r)] - \frac{1}{r^2}}} = \int_{r_{min}}^\infty \frac{s dr/r^2}{\sqrt{1 - \frac{V(r)}{E} - \frac{s^2}{r^2}}}.$$

Substitute $u = 1/r$:

$$\Rightarrow \;\psi = \int_0^{u_{max}} \frac{s du}{\sqrt{1 - \frac{V(1/u)}{E} - s^2u^2}}.$$

Use energy conservation to determine u_{max}:

$$E = \frac{1}{2}m\dot{r}^2 + \frac{\ell^2}{2mr^2} + V(r) = \frac{\ell^2}{2mr_{min}^2} + V(r_{min}) = Es^2u_{max}^2 + V(1/u_{max}).$$

$$\Rightarrow \; s^2u_{max}^2 + V(1/u_{max})/E = 1 \quad \Rightarrow \; u_{max} = u_{max}(s, E).$$

Scattering angle: $\theta(s, E) = \pi - 2\int_0^{u_{max}} \frac{s du}{\sqrt{1 - V(1/u)/E - s^2u^2}}.$

Total scattering cross section:

$$\sigma_T = \int \sigma(\theta) d\Omega = 2\pi \int_0^\pi \frac{s}{\sin\theta}\left|\frac{ds}{d\theta}\right| \sin\theta \, d\theta = 2\pi \int_0^{s_{max}} s\, ds.$$

Note: In quantum mechanics σ_T can be finite even if s_{max} is infinite.