Precession of the Perihelion [mln21]

Gravitation $\Rightarrow$ inverse-square force $\Rightarrow$ bounded orbits are closed (stationary ellipses with sun at one focus) $\Rightarrow$ apsidal angle: $\Delta\vartheta = \pi$.

Any slight deviation from the $1/r^2$-force law causes a precession of the perihelion in planetary orbits.

The perihelion of the planets Mercury, Venus, and Mars, was long observed to precess. The main cause is the presence of other planets in the solar system.

For Mercury the total precession observed is 531" per century, of which 43" are unexplained by many-body effects.

The residual 43" per century can be accounted for as a combination of relativistic effects:

- $\frac{1}{3}$ due to time dilation (special relativity),
- $\frac{1}{6}$ due to mass increase (special relativity),
- $\frac{1}{2}$ due to the fact that force is not instantaneous (general relativity).

In the framework of a pertubation calculation, the relativistic effects can be taken into account as a correction to the gravitational potential:

$$V(r) = -\frac{\kappa}{r} - \frac{\gamma}{r^3}, \qquad \kappa = GmM, \quad \gamma = \frac{G\ell^2 M}{c^2 m}.$$

Here m is the mass of the planet, M is the solar mass, G is the universal gravitational constant, ℓ is the angular momentum of the orbit, and c is the speed of light.

The angle $\delta\vartheta$ of precession per cycle can be calculated by different means including the following:

- perturbative correction to the orbital integral $\Rightarrow$ [mex165],
- perturbative solution of the orbital integral equation $\Rightarrow$ [mex166].

The result of both methods is $\delta\vartheta = 6\pi\left(\frac{GmM}{c\ell}\right)^2 \simeq \frac{6\pi GM}{c^2 a(1-e^2)}$.

The effect is enhanced when the semi-major axis a is small and the eccentricity e is large. Hence it is most prominent in Mercury's orbit.