Dynamics of Rigid Bodies

System with 6 degrees of freedom (3 translations and 3 rotations).

Equations of motion: \( \dot{p} = F^{(r)}, \quad \dot{L} = N^{(r)} \).

For the description of the rigid-body dynamics it is useful to introduce three coordinate systems:

- inertial coordinate system with axes \((X, Y, Z)\),
- coordinate system with axes \((x', y', z')\) parallel to \((X, Y, Z)\) and origin \(O\) fixed to some point of the rigid body,
- coordinate system with axes \((x, y, z)\) fixed to rigid body and with the same origin \(O\) as \((x', y', z')\).

Motion of rigid body: \( \mathbf{v}_a = \dot{\mathbf{R}} + \omega \times \mathbf{r}_a \).

- *Translational* motion of \((x', y', z')\) relative to \((X, Y, Z)\).
- *Rotational* motion of \((x, y, z)\) relative to \((x', y', z')\).

The optimal choice of the origin \(O\) is dictated by the circumstances:

- for freely rotating rigid bodies, the center of mass is the best choice;
- for rigid bodies rotating about at least one point fixed in the inertial system, one such fixed point is a good choice.

Analysis of rigid body motion:

- Solve the equations of motion in the coordinate system \((x, y, z)\). They are called *Euler’s equations*.
- Transform the solution to the coordinate system \((x', y', z')\) via *Eulerian angles* and from there to the inertial system \((X, Y, Z)\).