

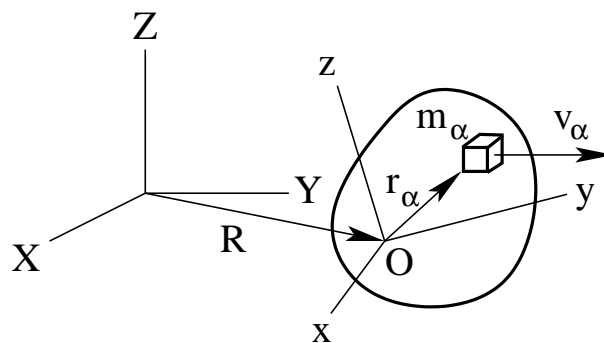
# Dynamics of Rigid Bodies [mln24]

System with 6 degrees of freedom (3 translations and 3 rotations).

Equations of motion:  $\dot{\mathbf{p}} = \mathbf{F}^{(e)}$ ,  $\dot{\mathbf{L}} = \mathbf{N}^{(e)}$  [mln2].

For the description of the rigid-body dynamics it is useful to introduce three coordinate systems:

- inertial coordinate system with axes  $(X, Y, Z)$ ,
- coordinate system with axes  $(x', y', z')$  parallel to  $(X, Y, Z)$  and origin  $O$  fixed to some point of the rigid body,
- coordinate system with axes  $(x, y, z)$  fixed to rigid body and with the same origin  $O$  as  $(x', y', z')$ .



Motion of rigid body:  $\mathbf{v}_\alpha = \dot{\mathbf{R}} + \vec{\omega} \times \mathbf{r}_\alpha$ .

- *Translational* motion of  $(x', y', z')$  relative to  $(X, Y, Z)$ .
- *Rotational* motion of  $(x, y, z)$  relative to  $(x', y', z')$ .

The optimal choice of the origin  $O$  is dictated by the circumstances:

- for freely rotating rigid bodies, the center of mass is the best choice;
- for rigid bodies rotating about at least one point fixed in the inertial system, one such fixed point is a good choice.

Analysis of rigid body motion:

- Solve the equations of motion in the coordinate system  $(x, y, z)$ . They are called *Euler's equations* [mln27].
- Transform the solution to the coordinate system  $(x', y', z')$  via *Eulerian angles* [msl25], [msl26] and from there to the inertial system  $(X, Y, Z)$ .