Driven Harmonic Oscillator I [mln28]

Equation of motion: $m\ddot{x} = -kx - \gamma\dot{x} + F_0 \cos\omega t \;\Rightarrow\; \ddot{x} + 2\beta\dot{x} + \omega_0^2 x = A\cos\omega t$.

Parameters: $\beta \doteq \gamma/2m, \quad \omega_0 \doteq \sqrt{k/m}, \quad A \doteq F_0/m$.

General solution: $x(t) = x_c(t) + x_p(t)$.

- $x_c(t)$: general solution of homogen. eq. (transients) $\Rightarrow$ [mln6].
- $x_p(t)$: particular solution of inhomogen. eq. (steady state) $\Rightarrow$ [mex180].

Steady-state oscillation: $x_p(t) = D\cos(\omega t - \delta)$

- amplitude: $D(\omega) = \dfrac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}}$,
- phase angle: $\delta(\omega) = \arctan \dfrac{2\omega\beta}{\omega_0^2 - \omega^2}$.

Maximum amplitude realized at $\left.\dfrac{dD(\omega)}{d\omega}\right|_{\omega_R} = 0$.

Amplitude resonance frequency: $\omega_R = \sqrt{\omega_0^2 - 2\beta^2}$ if $2\beta^2 < \omega_0^2$.

Average energy: $\langle E(\omega)\rangle = \langle T(\omega)\rangle + \langle V(\omega)\rangle = \dfrac{1}{4} m A^2 \dfrac{\omega^2 + \omega_0^2}{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}$.

$\langle E(\omega)\rangle$, $\langle T(\omega)\rangle$, $\langle V(\omega)\rangle$ are resonant at different frequencies $\Rightarrow$ [mex181].

Average power input: $\langle P(\omega)\rangle \doteq \langle F_0 \cos\omega t \cdot \dot{x}(t)\rangle \;\Rightarrow\;$ [mex182].

Quality factor: $\Rightarrow$ [mex183]

- driven oscillator: $Q \doteq 2\pi \dfrac{\text{average energy stored}}{\text{maximum energy input per period}}$,
- damped oscillator: $Q \doteq 2\pi \dfrac{\text{energy stored}}{\text{energy loss per period}}$.

For $\beta \ll \omega_0$ the width at half maximum of the power resonance curve is $\Delta\omega \simeq 2\beta$. Therefore, the quality factor is $Q \simeq \omega_0/\Delta\omega$.