

Newton's Law of Gravitation [mln3]

Gravitational potential of a mass distribution: $\Phi(\mathbf{r}) = -G \int d^3r' \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$;

$\rho(\mathbf{r})$: mass density;

$G \simeq 6.67 \times 10^{-11} \text{Nm}^2\text{kg}^{-2}$: universal gravitational constant.

Gravitational field (acceleration due to gravity):

$$\mathbf{g}(\mathbf{r}) = -\nabla\Phi(\mathbf{r}) = -G \int d^3r' \frac{\rho(\mathbf{r}')(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} \Rightarrow \Phi(\mathbf{r}) = - \int_{\mathbf{r}_0}^{\mathbf{r}} d\mathbf{r}' \cdot \mathbf{g}(\mathbf{r}')$$

Gravitational force on point mass m : $\mathbf{F}(\mathbf{r}) = m\mathbf{g}(\mathbf{r})$.

Gravitational potential energy of point mass m : $V(\mathbf{r}) = m\Phi(\mathbf{r})$.

Field equation: $-\nabla \cdot \mathbf{g}(\mathbf{r}) = \nabla^2\Phi(\mathbf{r}) = G \int d^3r' \rho(\mathbf{r}') \nabla \cdot \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} = 4\pi G\rho(\mathbf{r})$,

where we have used $\nabla \cdot \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^3} = 4\pi\delta(\mathbf{r} - \mathbf{r}')$.

Gauss' theorem: $\oint d\mathbf{A} \cdot \mathbf{g}(\mathbf{r}) = \int_V d^3r \nabla \cdot \mathbf{g}(\mathbf{r})$.

Gauss' law: $\oint d\mathbf{A} \cdot \mathbf{g}(\mathbf{r}) = -4\pi Gm_{in}$.

Gravitational self energy: $V_S = -\frac{1}{2}G \int d^3r \int d^3r' \frac{\rho(\mathbf{r})\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$.

Massive spheres:

$$\mathbf{F}_{12} = -\mathbf{F}_{21} = -G \frac{m_1 m_2}{R^2} \hat{\mathbf{r}},$$

$$\mathbf{g}(R) = \frac{\mathbf{F}_{12}}{m_2} = -G \frac{m_1}{R^2} \hat{\mathbf{r}},$$

$$\Phi(R) = - \int_{\infty}^R dr g(r) = -G \frac{m_1}{R}$$

$$V_2(R) = m_2\Phi(R) = -G \frac{m_1 m_2}{R}$$

