Feedback Control [mln33]

Consider the phase diagram of the plane pendulum as given in [msl8]. The upright rest position is an unstable equilibrium (hyperbolic fixed point).

Feedback control: Introduce a lateral motion of the pivot which is coupled to the instantaneous angular position and angular velocity in such a way that the upright rest position becomes a stable fixed point.

Displacement of pendulum bob along arc: $s = L\phi$.

Equation of motion: $m\ddot{s} = mg\sin\phi - m\ddot{w}\cos\phi$.

Horizontal displacement of pivot: $w(t)$.

Change of variables: $x_1 = \phi, x_2 = \dot{\phi}$.

Design of feedback: $\ddot{w} = c_1 x_1 + c_2 x_2$, where c_1, c_2 are controllable parameters.

Equation of motion with feedback:

$$\dot{x}_1 = x_2, \tag{1a}$$

$$\dot{x}_2 = \frac{c_1 x_1}{L}\cos x_1 + \frac{c_2 x_2}{L}\cos x_1 + \frac{g}{L}\sin x_1. \tag{1b}$$

Goal: Find the conditions for the control parameters c_1, c_2 which make the state $\phi = \dot{\phi} = 0$, i.e. $(x_1, x_2) = (0, 0)$ a stable equilibrium.