Geodesics [mln38]

The term *geodesic* originates from surveying the Earth's surface over distances so large that its curvature is significant.

Mathematical definition:
A *geodesic* is the shortest line between two points on any given surface.

Applications:

- Geodesics on a plane are straight lines [mex26], [mex117].
- Geodesics on a sphere lie on great circles [mex118].

Relation to dynamics:
Consider a particle of mass m that is constrained to move on a surface specified by a holonomic constraint $g(x, y, z) = 0$ and is not subject to any forces other than the forces of constraint. The path of such a particle consists of segments that are all geodesics.

Sketch of a proof: The potential energy V is identically zero and the energy E is conserved. Therefore the kinetic energy T, the speed v of the particle, and the Lagrangian $L = T - V$ are constants. Now consider Hamilton's principle for paths with constant L. The action J is then minimized if the time of travel, $t_2 - t_1$, is minimized, which, in turn, is the case on the shortest path, i.e. on a geodesic.

Clairaut's theorem:
Consider a surface of revolution described by cylindrical coordinates $z, \phi, r(z)$. Suppose a particle with mass m, constrained to move on that surface, is launched with a speed v_0 at $\phi = z = 0$ in a direction at an angle α_0 from the meridian. (The intersection between the surface and a plane through its axis produces two meridians.) From the conservation of kinetic energy and the conservation of angular momentum around the axis it follows that $r \sin\alpha = \text{const}$ holds along the path of the particle.

Applications:

- Dynamical trap without potential energy [mex119].
- Vertical range of particle sliding inside cone [mex120].