Newtonian mechanics in the presence of holonomic constraints [mln5]

Equations of motion: $m_i \ddot{\mathbf{r}}_i = \mathbf{F}_i + \mathbf{Z}_i, \quad i = 1, \ldots, N.$

$\mathbf{F}_i$: applied forces (known),

$\mathbf{Z}_i$: forces of constraint (unknown).

Equations of constraint: $f_j(\mathbf{r}_1, \ldots, \mathbf{r}_N) = 0,\ j = 1, \ldots, k$ (scleronomic).

The solution of this dynamical problem within the framework of Newtonian mechanics proceeds as follows:

- The number of degrees of freedom is reduced to $3N - k$.
- The number of equations of motion is $3N$ with $6N$ unknowns $(x_i, y_i, z_i, Z_{ix}, Z_{iy}, Z_{iz}),\ i = 1, \ldots, N$.
- The geometrical restrictions imposed by the constraints on the orbit yield $3N$ additional relations between the unknowns. Among them are the k equations of constraint.
- A unique solution depends on $6N$ initial conditions. Because of the constraints, the number of independent initial conditions is smaller than $6N$.
- The reduction of the number of degrees of freedom from $3N$ to $3N - k$ can be taken into account by introducing $n = 3N - k$ generalized coordinates $q_1, \ldots, q_n$ such that the functions $\mathbf{r}_i(q_1, \ldots, q_n),\ i = 1, \ldots, N$ satisfy the equations of constraint.
- The number of unknowns is thus reduced to $6N - k$. The number of equations of motion stays at $3N$. The number of additional relations due to the constraints is reduced to $3N - k$.

Examples:

- Plane pendulum I [mex132]
- Heavy particle sliding inside cone I [mex133]