Time Dilation Paradox [mln52]

Consider two reference frames S and S' in relative motion. An observer in S determines that time in S' is slowed, whereas an observer in S' determines that time in S is slowed. How can both observers be right?

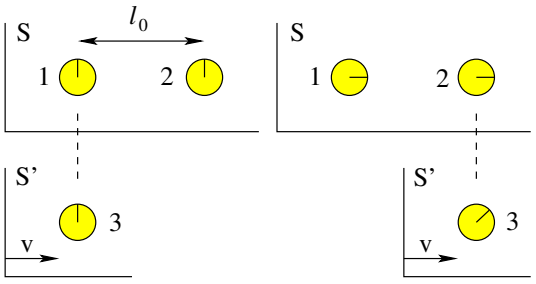
Clocks 1 and 2 are synchronized in S for all times.
Clocks 1 and 3 are synchronized at $t = 0$.

View from frame S:

Proper distance between clocks 1 and 2: ℓ_o.

Reading of clock 2 when clock 3 arrives there: $t = \ell_0/v$.

Time elapsed in S': $t' = (\ell_0/v)\sqrt{1 - v^2/c^2}$.



View from frame S':

Distance between clocks 1 and 2: $\ell' = \ell_0\sqrt{1 - v^2/c^2}$.

Reading of clock 3 when it reaches clock 2: $t' = \ell'/v = (\ell_0/v)\sqrt{1 - v^2/c^2}$.

Time elapsed in S: $\Delta t = t'\sqrt{1 - v^2/c^2} = (\ell_0/v)(1 - v^2/c^2)$.

Initial reading of clock 2 as viewed from S': $t_i = \ell_0 v/c^2$.

Final reading of clock 2: $t_i + \Delta t = \ell_0/v$.