Longitudinal Doppler Effect [mln57]

Sound:

Transmitter T and receiver R moving with relative velocity v toward each other. Sound wave propagates with velocity v_s through medium.
Transmitter emits $N_T = \nu_T \Delta t$ cycles at frequency ν_T in time Δt.

(a) Receiver at rest in medium:

Advance toward R of wave in time Δt: $N_T \lambda = (v_s - v)\Delta t$.
Frequency received: $\nu_R = \dfrac{v_s}{\lambda} = \dfrac{N_T}{\Delta t}\dfrac{v_s}{v_s - v} = \dfrac{\nu_T}{1 - v/v_s}$.

(b) Transmitter at rest in medium:

Distance travelled by wave relative to receiver in Δt: $(v_s + v)\Delta t$.
Distance travelled by wave relative to transmitter in Δt: $v_s \Delta t$.
Receiver detects $N_R = \nu_R \Delta t$ cycles at frequency ν_R in time Δt.

Relation between N_R and N_T: $N_R = N_T \dfrac{v_s + v}{v_s}$.
Frequency received: $\nu_R = \dfrac{N_R}{\Delta t} = \dfrac{N_T}{\Delta t}\dfrac{v_s + v}{v_s} = \nu_T(1 + v/v_s)$.

Light:

Transmitter T and receiver R moving with relative velocity v toward each other. Light wave propagates with velocity c relative to T and relative to R.

Transmitter emits $N_T = \nu_T \Delta t_T$ cycles at frequency ν_T in time Δt_T.
Advance toward R of wave in time Δt_R: $N_T \lambda = (c - v)\Delta t_R$.
Time dilation: $\Delta t_R = \Delta t_T / \sqrt{1 - v^2/c^2}$.
Frequency received: $\nu_R = \dfrac{c}{\lambda} = \nu_T \sqrt{\dfrac{1 + v/c}{1 - v/c}}$.