Coordinate Transformations [mln58]

Galilei transformation:

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t.$$

Lorentz transformation:

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}.$$

- Check time dilation: $t'_2 - t'_1 = \dfrac{t_2 - t_1}{\sqrt{1 - v^2/c^2}}$ for $x_1 = x_2$.
  Proper time interval: $\Delta\tau \doteq t_2 - t_1$.
- Check length contraction: $x'_2 - x'_1 = \dfrac{x_2 - x_1}{\sqrt{1 - v^2/c^2}}$ for $t_1 = t_2$
  Proper length: $\ell_0 \doteq x'_2 - x'_1$.
- Check relativity of simultaneity: $t' = t(x) = \dfrac{t - vx/c^2}{\sqrt{1 - v^2/c^2}}$.
  Clocks synchronized at $x = x' = 0$.

Longitudinal velocity addition:

Substitute $x = v_p t$ and $x' = v'_p t'$ into transformation equations.

- Nonrelativistic: $v_p = v'_p + v \quad \Rightarrow \ v'_p = v_p - v$.
- Relativistic: $v_p = \dfrac{v'_p + v}{1 + v'_p v/c^2} \quad \Rightarrow \ v'_p = \dfrac{v_p - v}{1 - v_p v/c^2}$.

Transverse velocity addition:

Set $y' = u'_p t'$ and $y = u_p t$ and $x = v_p t$, $x' = v'_p t'$.
Then use $y = y'$ and time dilation.

- Nonrelativistic: $u'_p = u_p$.
- Relativistic: $u'_p = \dfrac{u_p\sqrt{1 - v^2/c^2}}{1 - v_p v/c^2} \quad \Rightarrow \ u_p = \dfrac{u'_p\sqrt{1 - v^2/c^2}}{1 + v'_p v/c^2}$.