# Observing Transverse Motion of Meter Stick [mln60]

Consider a meter stick aligned in $x$-direction of frame $S$ and moving with velocity $v_y$. Frame $S'$ moves in $x$-direction with velocity $v$ relative to $S$. The center of the stick passes the point $x = x' = 0$, $y = y' = 0$ at $t = t' = 0$.

Viewed from $S$ the two ends of the stick reach $y = y' = 0$ simultaneously.

Viewed from $S'$ the right end of the stick goes through $y = y' = 0$ before the left end does. This is a consequence of the relativity of simultaneity. Hence the stick appears tilted in $S'$ as shown.

Event 1: right end of stick as it crosses $x'$-axis.

$$x'_1 = \frac{x_1 - vt_1}{\sqrt{1 - v^2/c^2}} = \frac{(0.5\text{m})}{\sqrt{1 - v^2/c^2}}, \quad y'_1 = 0.$$

$$t'_1 = \frac{t_1 - vx_1/c^2}{\sqrt{1 - v^2/c^2}} = -\frac{(0.5\text{m})v/c^2}{\sqrt{1 - v^2/c^2}}.$$

Event 2: center of stick as it crosses $x'$-axis.

$$x'_2 = 0, \quad y'_2 = 0, \quad t'_2 = 0.$$

Event 3: right end of stick as center crosses $x'$-axis.

$$x'_3 = x'_1 + v'_x(t'_3 - t'_1), \quad y'_3 = y'_1 + v'_y(t'_3 - t'_1), \quad t'_3 = 0.$$

Velocity of stick in $S'$: $v'_x = -v, \quad v'_y = v_y\sqrt{1 - v^2/c^2}$.

$$\Rightarrow \; x'_3 = (0.5\text{m})\sqrt{1 - v^2/c^2}, \quad y'_3 = (0.5\text{m})\frac{vv_y}{c^2}.$$

Tilt angle: $\tan\phi = \dfrac{y'_3}{x'_3} = \dfrac{vv_y/c^2}{\sqrt{1 - v^2/c^2}}$.