Relativistic Momentum

Ansatz for relativistic momentum: \( p = m(v)v \).

Two particles with equal masses \( m \) as measured when at rest are undergoing an inelastic collision as shown in the lab frame \( S \) and in the frame \( S' \) moving with velocity \( v \) to the right.

1. Relation between \( v \) and \( \bar{v} \) from \([mln58]\) and symmetry:
   \[
   \bar{v} = -\bar{v} + v \quad \Rightarrow \quad v = \frac{2\bar{v}}{1 + \bar{v}^2/c^2}.
   \]

2. Conservation of total momentum:
   \[ m(v)v + m(0)0 = M(\bar{v})\bar{v}. \]

3. Lorentz invariance of momentum conservation implies \([mex221]\):
   \[ M(\bar{v}) = m(v) + m(0). \]

Relativistic mass from 1.–3. \([mex222]\):
\[
 m(v) = \frac{m_0}{\sqrt{1 - v^2/c^2}},
\]
where \( m_0 = m(0) \) is called the rest mass.

Relativistic momentum:
\[
 p = \frac{m_0v}{\sqrt{1 - v^2/c^2}}.
\]