Relativistic Energy II [mln65]

Relativistic adaptation of Newton's equation of motion:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}, \qquad \mathbf{p} = \frac{m_0\mathbf{v}}{\sqrt{1-v^2/c^2}}.$$

Conservative force: $\mathbf{F} = -\nabla U$.

Work and potential energy: $W_{12} = \int_1^2 d\mathbf{r}\cdot\mathbf{F} = -(U_2 - U_1)$.

Work and relativistic energy:

$$W_{12} = \int_1^2 dt\,\mathbf{v}\cdot\frac{d}{dt}\left(\frac{m_0\mathbf{v}}{\sqrt{1-v^2/c^2}}\right) = \int_1^2 dt\frac{d}{dt}\left(\frac{m_0c^2}{\sqrt{1-v^2/c^2}}\right) = T_2 - T_1.$$

Energy conservation: $T_1 + U_1 = T_2 + U_2$.

Space-time four-vector: $x_\mu \doteq (ct, x_1, x_2, x_3)$.

Energy-momentum four-vector: $p_\mu \doteq (E/c, p_1, p_2, p_3)$.

Lorentz transformation:

$$x_1' = \frac{x_1 - vt}{\sqrt{1-v^2/c^2}}, \quad x_2' = x_2, \quad x_3' = x_3, \quad t' = \frac{t-(v/c^2)x_1}{\sqrt{1-v^2/c^2}}.$$

$$p_1' = \frac{p_1 - (v/c^2)E}{\sqrt{1-v^2/c^2}}, \quad p_2' = p_2, \quad p_3' = p_3, \quad E' = \frac{E - vp_1}{\sqrt{1-v^2/c^2}}.$$

Transformation of radiant energy ($p_1 = E/c$): $E' = E\sqrt{\dfrac{1-v/c}{1+v/c}}$.

Invariant quantity: $E^2/c^2 - \mathbf{p}^2 = m_0^2c^2$.

Relativistic energy-momentum relation: $E = \sqrt{\mathbf{p}^2c^2 + m_0^2c^4}$.

- Nonrelativistic limit: $E \simeq m_0c^2 + \dfrac{\mathbf{p}^2}{2m_0}$.
- Ultrarelativistic limit: $E \simeq \mathbf{p}c$.