Consider virtual displacements $\delta r_i$:

- they are infinitesimal;
- they satisfy the equations of constraint;
- they are instantaneous ($\delta t = 0$).

For scleronomic constraints, the paths of real and virtual displacements are the same; for rheonomic constraints, they differ.

Newton’s equations of motion: $m_i \ddot{r}_i = F_i + Z_i, \quad i = 1, \ldots, N$.

D’Alembert’s principle: $\sum_{i=1}^{N} Z_i \cdot \delta r_i = 0$.

The forces of constraint perform zero net work.

A consequence of D’Alembert’s principle is D’Alembert’s equation: $\sum_{i=1}^{N} (m_i \ddot{r}_i - F_i) \cdot \delta r_i = 0$.

It does no longer contain the forces of constraint.

Transformation to independent (generalized) coordinates,

$$\sum_{j=1}^{3N-k} \left[ \sum_{i=1}^{N} (m_i \ddot{r}_i - F_i) \cdot \frac{\partial r_i}{\partial q_j} \right] \delta q_j = 0,$$

results in $3N - k$ equations of motion, one for each remaining degree of freedom:

$$\sum_{i=1}^{N} (m_i \ddot{r}_i - F_i) \cdot \frac{\partial r_i}{\partial q_j} = 0, \quad j = 1, \ldots, 3N - k.$$

Applications:

- Plane pendulum II [mex134]
- Heavy particle sliding inside cone II [mex135]