

# Solution by Separation of Variables [mln72]

## Velocity-dependent attenuation

Equation of motion:  $m\ddot{x} = F$  with  $F = f(\dot{x})g(t)$ .

$$\Rightarrow m \frac{dv}{dt} = f(v)g(t) \quad \Rightarrow m \int_{v_0}^v \frac{dv}{f(v)} = \int_0^t dt g(t).$$

Solve for  $v(t)$ . Then calculate  $x(t) = \int_0^t dt v(t)$ .

Applications: [mex15], [mex16], [mex230].

## Rocket motion

Instantaneous momentum of rocket:  $p_R(t) = m(t)v(t)$ .

Momentum increment of exhaust gases:  $\Delta p_E(t) = -[u - v(t)](-\Delta m)$ .

Speed of exhaust gases relative to rocket:  $u$ .

Equation of motion:  $\dot{p}_R + \dot{p}_E = F_{\text{ext}}$ .

$$\Rightarrow m\dot{v} + \dot{m}v - (u - v)(-\dot{m}) = F_{\text{ext}}.$$

$$\Rightarrow m\dot{v} + \dot{m}u = F_{\text{ext}}.$$

Rocket motion in free space:

$$F_{\text{ext}} = 0 \quad \Rightarrow \quad \frac{dv}{u} = -\frac{dm}{m} \quad v(t) = u \ln \frac{m_0}{m(t)}.$$

Applications: [mex17], [mex18], [mex229].

Photon rocket [mex223].