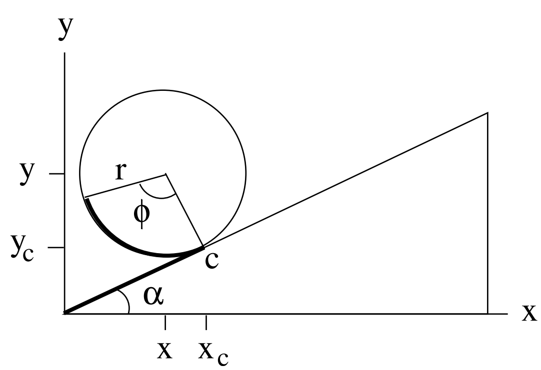
Disk Rolling Along Incline [mln76]

Rigid body in (x, y, z)-space has six degrees of freedom.

Reduction to three degrees of freedom by implicit constraints:

- Translational motion constrained to (x, y)-plane (down one).
- Rotation constrained to plane of disk (down another two).

Coordinates of free disk in plane:
center-of-mass position (x, y) and orientation (ϕ).



The requirement that the disk roll along the incline amounts to two additional constraints:

$$x = x_c - r\sin\alpha, \quad y = y_c + r\cos\alpha,$$

where $x_c = r\phi\cos\alpha$, $y_c = r\phi\sin\alpha$.

$$\Rightarrow \; x = r\phi\cos\alpha - r\sin\alpha, \quad y = r\phi\sin\alpha + r\cos\alpha.$$

The position and orientation of the rolling disk can be described by one independent variable (ϕ). The rolling disk has one degree of freedom left.

Differential form of constraint (in the context of [mln37]):

$$dx = r\cos\alpha\, d\phi, \quad dy = r\sin\alpha\, d\phi$$

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