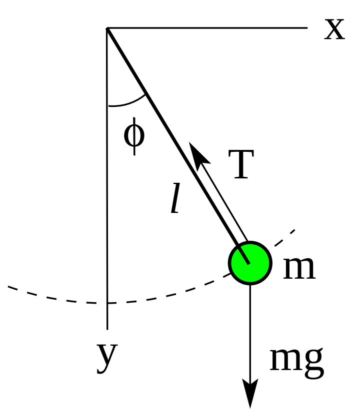
Simple Applications of Lagrangian Mechanics [mln77]

☐ **Plane pendulum:** one degree of freedom.



Lagrangian: $L = T - V = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2\right) + mgy.$

Generalized coordinate: $x = \ell \sin\phi,\ y = \ell\cos\phi.$

$\Rightarrow\ \ L = \frac{1}{2}m\ell^2\dot{\phi}^2 + mg\ell\cos\phi.$

Lagrange equation: $\frac{d}{dt}\frac{\partial L}{\partial\dot{\phi}} - \frac{\partial L}{\partial\phi} = 0.$

$\frac{\partial L}{\partial\phi} = -mgl\sin\phi, \quad \frac{\partial L}{\partial\dot{\phi}} = m\ell^2\dot{\phi}, \quad \frac{d}{dt}\frac{\partial L}{\partial\dot{\phi}} = m\ell^2\ddot{\phi}.$

$\Rightarrow\ \ \ddot{\phi} + \frac{g}{\ell}\sin\phi = 0.$

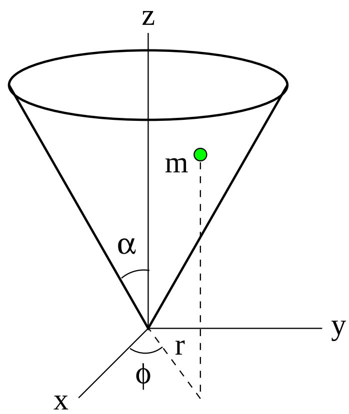
☐ **Particle sliding inside cone:** two degrees of freedom.

Lagrangian: $L = T - V = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right) - mgz.$

Generalized coordinates: $x = r\cos\phi,\ y = r\sin\phi,\ z = r\cot\alpha.$

$\Rightarrow\ \ L = \frac{1}{2}m\left[\dot{r}^2\left(1+\cot^2\alpha\right) + r^2\dot{\phi}^2\right] - mgr\cot\alpha.$

Lagrange equations:



$\frac{\partial L}{\partial\phi} = 0, \quad \frac{\partial L}{\partial\dot{\phi}} = mr^2\dot{\phi}, \quad \frac{d}{dt}\frac{\partial L}{\partial\dot{\phi}} = 2mr\dot{r}\dot{\phi} + mr^2\ddot{\phi}.$

$\Rightarrow\ \ 2\dot{r}\dot{\phi} + r\ddot{\phi} = 0.$

$\frac{\partial L}{\partial r} = mr\dot{\phi}^2 - mg\cot\alpha, \quad \frac{\partial L}{\partial\dot{r}} = m\dot{r}\left(1+\cot^2\alpha\right),$

$\frac{d}{dt}\frac{\partial L}{\partial\dot{r}} = m\ddot{r}\left(1+\cot^2\alpha\right).$

$\Rightarrow\ \ \ddot{r}\left(\tan\alpha + \cot\alpha\right) - r\dot{\phi}^2\tan\alpha + g = 0.$