Use of Cyclic Coordinates [mln84]

Lagrangian mechanics:

Lagrangian: $L(q_1, \ldots, q_{n-1}; \dot{q}_1, \ldots, \dot{q}_n)$.

Cyclic coordinate q_n: $\Rightarrow \dfrac{\partial L}{\partial q_n} = 0 \quad \Rightarrow \dfrac{d}{dt}\dfrac{\partial L}{\partial \dot{q}_n} = 0.$

Conserved quantity: $\dfrac{\partial L}{\partial \dot{q}_n} \doteq \beta_n(q_1, \ldots, q_{n-1}; \dot{q}_1, \ldots, \dot{q}_n) = \text{const.}$

Eliminate $\dot{q}_n = \dot{q}_n(q_1, \ldots, q_{n-1}; \dot{q}_1, \ldots, \dot{q}_{n-1}; \beta_n)$ as independent variable.

Do not substitute $\dot{q}_n(q_1, \ldots, q_{n-1}; \dot{q}_1, \ldots, \dot{q}_{n-1}; \beta_n)$ into Lagrangian.
Substitute $\dot{q}_n(q_1, \ldots, q_{n-1}; \dot{q}_1, \ldots, \dot{q}_{n-1}; \beta_n)$ into Routhian instead.

Routhian: $R(q_1, \ldots, q_{n-1}; \dot{q}_1, \ldots, \dot{q}_{n-1}; \beta_n) = L - \beta_n \dot{q}_n$.

Equations of motion: $\dfrac{\partial R}{\partial q_i} - \dfrac{d}{dt}\dfrac{\partial R}{\partial \dot{q}_i} = 0, \quad i = 1, \ldots, n-1.$

Supplement: $q_n(t) = -\displaystyle\int dt\, \frac{\partial R}{\partial \beta_n}.$

Hamiltonian mechanics:

Hamiltonian: $H(q_1, \ldots, q_{n-1}; p_1, \ldots, p_n)$.

Cyclic coordinate q_n: $\Rightarrow \dfrac{\partial H}{\partial q_n} = 0 \quad \Rightarrow \dot{p}_n = 0.$

Conserved quantity: $p_n \doteq \alpha_n = \text{const.}$

Reduced Hamiltonian: $H(q_1, \ldots, q_{n-1}; p_1, \ldots, p_{n-1}; \alpha_n)$.

Angular frequency: $\omega_n \doteq \dot{q}_n(q_1, \ldots, q_{n-1}; p_1, \ldots, p_{n-1}; \alpha_n) = \dfrac{\partial H}{\partial \alpha_n}.$

Equations of motion: $\dot{q}_i = \dfrac{\partial H}{\partial p_i}, \quad \dot{p}_i = -\dfrac{\partial H}{\partial q_i}, \quad i = 1, \ldots, n-1.$

Supplement: $q_n(t) = \displaystyle\int dt\, \omega_n(t).$