Velocity-Dependent Potential Energy [mln85]

Lagrange equations in raw form from [mln8]:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_j} - \frac{\partial T}{\partial q_j} - Q_j = 0, \quad j = 1, \ldots, n,$$

- kinetic energy: $T(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t)$,
- generalized forces: $Q_j(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t)$.

The standard form of the Lagrange equations,

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = 0, \quad j = 1, \ldots, 3n,$$

can be inferred from a Lagrangian $L(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t)$ under the following circumstances:

(a) If the generalized forces can be derived from a position-dependent potential energy$V(q_1, \ldots, q_n; t)$,

$$Q_j(q_1, \ldots, q_n; t) = -\frac{\partial V}{\partial q_j},$$

then the Lagrangian is $L = T - V$.

(b) If the generalized forces can be derived from a velocity-dependent potential energy $U(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t)$,

$$Q_j(q_1, \ldots, q_n; \dot{q}_1, \ldots, \dot{q}_n; t) = -\frac{\partial U}{\partial q_j} + \frac{d}{dt}\frac{\partial U}{\partial \dot{q}_j},$$

then the Lagrangian is $L = T - U$.

Hamiltonian derived from the Lagrangian via Legendre transform:

$$H(q_1, \ldots, q_n; p_1, \ldots, p_n; t) = \sum_{j=1}^{n} p_j \dot{q}_j - L, \quad p_j = \frac{\partial L}{\partial \dot{q}_j}.$$

Examples of velocity-dependent potential energy:

- Lorentz force [mln86],
- velocity-dependent central force [mex76].