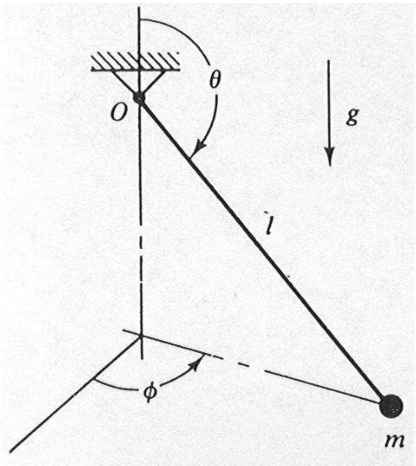
Integrability as a Contingent Property [mln99]

In autonomous Hamiltonian systems with two (or more) degrees of freedom, integrability is the exception rather than the rule.

Prototypical examples of integrable and nonintegrable systems emerge from modifications of the plane pendulum [mln98] by adding a degree of freedom.

Integrable system:

spherical pendulum



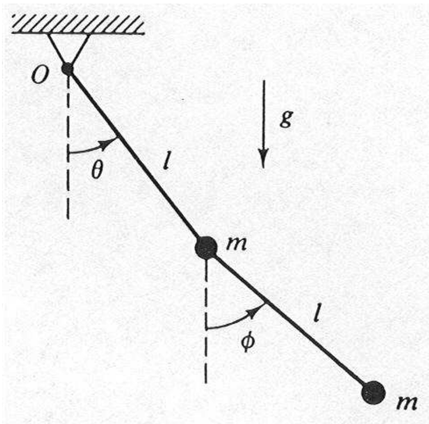
4D phase space: $(\theta, \phi, p_\theta, p_\phi)$

3D hypersurface: $E = \text{const.}$
3D hypersurface: $p_\phi = \text{const.}$

Trajectories are nonintersecting on 2D invariant torus $E = \text{const.} \cap p_\phi = \text{const.}$

Nonintegrable system:

plane double pendulum



4D phase space: $(\theta, \phi, p_\theta, p_\phi)$

3D hypersurface: $E = \text{const.}$
No second invariant subspace.

Trajectories are nonintersecting on 3D energy hypersurface.

Types of trajectories:

Integrable system:

- periodic,
- quasiperiodic.

Nonintegrable system:

- periodic,
- quasiperiodic,
- chaotic.