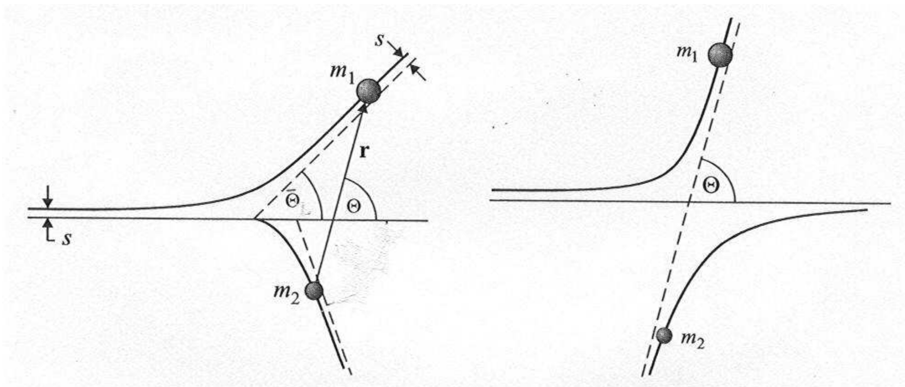
Scattering angle in the laboratory frame [msl3]

The scattering experiment is performed in the **laboratory frame**.

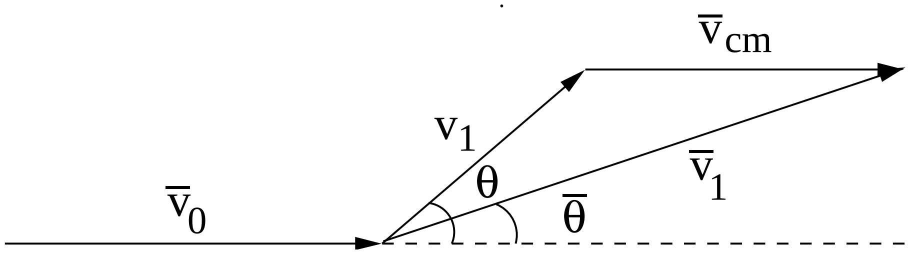
- observed scattering angle: $\bar{\theta}$,
- observed scattering cross section: $\bar{\sigma}(\bar{\theta})$,
- projectile of mass m_1 and target of mass m_2.

The theoretical analysis is performed in the **center-of-mass frame**:

- problem reduced to one degree of freedom,
- total mass $M = m_1 + m_2$,
- reduced mass $m = m_1 m_2/(m_1 + m_2)$,
- calculated scattering angle: θ,
- calculated scattering cross section: $\sigma(\theta)$.



Task #1: establish the relation between θ and $\bar{\theta}$.



$$m_1 \bar{\mathbf{v}}_0 = (m_1 + m_2)\bar{\mathbf{v}}_{cm} \quad \Rightarrow \; \bar{\mathbf{v}}_{cm} = \frac{m_1}{m_1 + m_2}\,\bar{\mathbf{v}}_0 = \frac{m}{m_2}\,\bar{\mathbf{v}}_0.$$

$$\bar{v}_1 \sin\bar{\theta} = v_1 \sin\theta, \quad \bar{v}_1 \cos\bar{\theta} = v_1 \cos\theta + \bar{v}_{cm}.$$

Relative velocity after collision: $\mathbf{v} = \bar{\mathbf{v}}_2 - \bar{\mathbf{v}}_1 = \mathbf{v}_2 - \mathbf{v}_1$ (frame-independent).

Linear momentum in center-of-mass frame: $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$.

$$\Rightarrow \ \mathbf{v}_1 = -\frac{m_2}{m_1+m_2}\mathbf{v}, \quad \mathbf{v}_2 = \frac{m_1}{m_1+m_2}\mathbf{v} \quad \Rightarrow \ m_1 v_1 = mv.$$

$$\Rightarrow \ \tan\bar{\theta} = \frac{v_1 \sin\theta}{v_1\cos\theta + \bar{v}_{cm}} = \frac{\sin\theta}{\cos\theta+\rho}, \quad \rho = \frac{m}{m_2}\frac{\bar{v}_0}{v_1} = \frac{m_1}{m_2}\frac{\bar{v}_0}{v}.$$

$$\Rightarrow \ \cos\theta = -\rho(1-\cos^2\bar{\theta}) + \cos\bar{\theta}\sqrt{1-\rho^2(1-\cos^2\bar{\theta})}.$$

Elastic scattering: $T = \frac{1}{2}m\bar{v}_0^2 = \frac{1}{2}mv^2$ (in center-of-mass frame)

$$\Rightarrow \ \bar{v}_0 = v \quad \Rightarrow \ \rho = m_1/m_2.$$

Task #2: establish the relation between σ and $\bar{\sigma}$.

Number of particles scattered into infinitesimal solid angle:
$2\pi I\sigma(\theta)\sin\theta|d\theta| = 2\pi I\bar{\sigma}(\bar{\theta})\sin\bar{\theta}|d\bar{\theta}|$.

$$\Rightarrow \ \bar{\sigma}(\bar{\theta}) = \sigma(\theta)\,\frac{\sin\theta}{\sin\bar{\theta}}\left|\frac{d\theta}{d\bar{\theta}}\right| = \sigma(\theta)\,\left|\frac{d\cos\theta}{d\cos\bar{\theta}}\right|.$$

$$\Rightarrow \ \bar{\sigma}(\bar{\theta}) = \sigma(\theta)\left[2\rho\cos\bar{\theta} + \frac{1+\rho^2\cos(2\bar{\theta})}{\sqrt{1-\rho^2\sin^2\bar{\theta}}}\right].$$

Special case: elastic scattering between particles of equal mass:

$m_1 = m_2 \quad \Rightarrow \ \cos\theta = \cos(2\bar{\theta}) \quad \Rightarrow \ \bar{\theta} = \frac{\theta}{2}, \quad \bar{\sigma}(\bar{\theta}) = 4\cos\frac{\theta}{2}\,\sigma(\theta).$