

[mex109] Fierce competition versus mild competition

Two species of animals vie for the same food source. In isolation each species grows logistically. Through interaction they impede each other's growth. The equations of motion for the two populations N_1, N_2 ,

$$\dot{N}_1 = rN_1 \left(1 - \frac{N_1}{K}\right) - \alpha N_1 N_2, \quad \dot{N}_2 = sN_2 \left(1 - \frac{N_2}{L}\right) - \beta N_1 N_2,$$

depend on six parameters: r, s are the per-capita growth rates, K, L the carrying capacities for the two populations N_1, N_2 , respectively, and α, β are the adverse impact parameters of the competitor population. Consider the two cases (i) $K = L = 2, r = s = 1, \alpha = \beta = 1$, and (ii) $K = L = 1, r = s = 2, \alpha = \beta = 1$.

Determine the nature of all four fixed points in each case as previously identified in [mex108]. What conclusions can be drawn from these results about the nature of the competition between the two species?

Solution: