Consider a particle of mass $m$ moving along the $x$-axis under the influence of a conservative force described by a potential energy function $V(x)$ which has a smooth maximum at $x = \bar{x}$ with curvature $|V''(\bar{x})| = k$.

(a) Find the slope of the tangent lines to the separatrix at the resulting hyperbolic fixed point $(\bar{x}, 0)$ in the $(x, \dot{x})$-plane.

(b) Calculate the time it takes the particle to move between two points $x_1$ and $x_2$ very close to the hyperbolic point $(x = \bar{x}, \dot{x} = 0)$ on the separatrix.

Solution: