

**[mex111] Separatrix tangent lines at hyperbolic point**

Consider a particle of mass  $m$  moving along the  $x$ -axis under the influence of a conservative force described by a potential energy function  $V(x)$  which has a smooth maximum at  $x = \bar{x}$  with curvature  $|V''(\bar{x})| = k$ .

- (a) Find the slope of the tangent lines to the separatrix at the resulting hyperbolic fixed point  $(\bar{x}, 0)$  in the  $(x, \dot{x})$ -plane.
- (b) Calculate the time it takes the particle to move between two points  $x_1$  and  $x_2$  very close to the hyperbolic point  $(x = \bar{x}, \dot{x} = 0)$  on the separatrix.

**Solution:**