Consider a surface of revolution with cylindrical coordinates $z, \phi, r(z) = (1 + z^2)^{-1}$. A particle of mass $m$ is constrained to move on that surface without friction. It is launched at $z = 0$ with a speed $v_0$ in a direction at $45^\circ$ relative to the meridian. Find the maximum value of $|z|$ the particle reaches along its trajectory.

Solution: