Consider the 2D harmonic oscillator with kinetic energy $T = \frac{1}{2} m (\dot{x}^2 + \dot{y}^2)$ and potential energy $V = \frac{1}{2} k (x^2 + y^2)$. (a) Express the Lagrangian of this system in polar coordinates $r, \theta$. (b) Identify the cyclic coordinate and construct the Routhian function which eliminates the cyclic coordinate. (c) Derive the equation of motion for the noncyclic coordinate and an integral expression for the cyclic coordinate.

**Solution:**