Consider a system of two blocks of mass $m$ attached by springs of stiffness $k$ to each other and to a rigid wall. The blocks can slide without friction along the $x$-axis. When the springs are relaxed, the blocks are at the positions $x_1 = a$ and $x_2 = 2a$. (a) Find the Lagrangian $L(q_1, q_2, \dot{q}_1, \dot{q}_2)$ for the system, where $q_1, q_2$ are the displacements of the two blocks from their equilibrium positions. (b) Find the angular frequencies $\omega_1, \omega_2$ of the two normal modes by solving the characteristic equation. (c) Find the amplitude ratios $A_1^{(k)}/A_2^{(k)}$, $k = 1, 2$ for the two normal modes.

Solution: