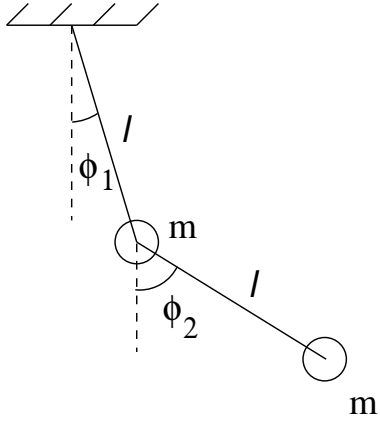


[mex124] Small oscillations of the double pendulum

Consider a plane double pendulum consisting of two equal point masses m and two rods of negligible mass and equal lengths ℓ . The Lagrangian $L(\phi_1, \phi_2, \dot{\phi}_1, \dot{\phi}_2)$ of this system is known from [mex20]. (a) Expand $L(\phi_1, \phi_2, \dot{\phi}_1, \dot{\phi}_2)$ to quadratic order in the dynamical variables and derive the Lagrange equations from it. They describe the small oscillations about the stable equilibrium position. (b) Find the angular frequencies ω_1, ω_2 of the two normal modes by solving the characteristic equation. (c) Find the amplitude ratios $A_1^{(k)}/A_2^{(k)}$, $k = 1, 2$ for the two normal modes.



Solution: