

**[mex127] Robustness of apsidal angles**

(a) Given the result of [mex126], namely that nearly circular orbits at radius  $r = R$  of a central force potential  $V(r)$  have apsidal angle  $\Delta\vartheta = \pi\sqrt{V'(R)/[3V'(R) + RV''(R)]}$ , show that the only cases for which this apsidal angle is independent of the radius are the power-law potentials  $V(r) = -\kappa/r^\alpha$ ,  $\alpha < 2$ ,  $\alpha \neq 0$  and the logarithmic potential  $V(r) = \kappa \ln r$ . (b) Show that the value of the apsidal angle is  $\Delta\vartheta = \pi/\sqrt{2 - \alpha}$ , where the value  $\alpha = 0$  pertains to the logarithmic potential.

**Solution:**