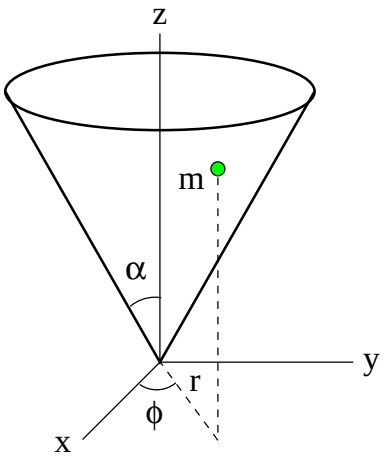
[mex133] Heavy particle sliding inside cone I

Consider a conical surface with vertical axis (z) and apex with angle 2α at the bottom in a uniform gravitational field g. A particle of mass m is free to slide on the inside of the cone.
(a) State the three equations of motion for the Cartesian coordinates x, y, z in terms of the known applied force $m\mathbf{g}$ and the unknown force of constraint $\mathbf{Z}$ (normal force).
(b) Derive three additional relations between the six unknowns x, y, z, Z_x, Z_y, Z_z geometrically from the constraint.
(c) Introduce cylindrical coordinates r, ϕ, z and derive from the six equations established previously the following two equations of motion for the two independent generalized coordinates:

$$2\dot{r}\dot{\phi} + r\ddot{\phi} = 0, \quad (\tan\alpha + \cot\alpha)\ddot{r} - r\dot{\phi}^2 \tan\alpha + g = 0.$$



Solution: