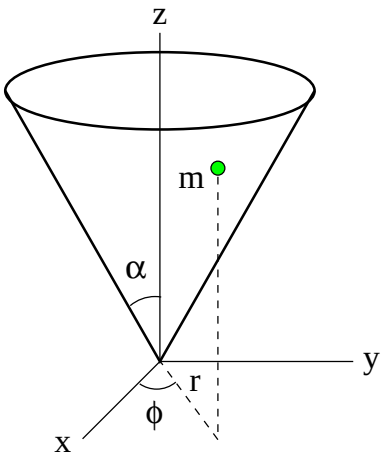
[mex135] Heavy particle sliding inside cone II

Consider a conical surface with vertical axis (z) and apex with angle 2α at the bottom in a uniform gravitational field g. A particle of mass m is free to slide on the inside of the cone. Derive the equations of motion $2\dot{r}\dot{\phi} + r\ddot{\phi} = 0$, $(\tan\alpha + \cot\alpha)\ddot{r} - r\dot{\phi}^2 \tan\alpha + g = 0$ for the two independent generalized coordinates ϕ, r from D'Alembert's equation,

$$(m\ddot{\mathbf{r}} - m\mathbf{g}) \cdot \delta\mathbf{r} = 0, \quad \mathbf{r} = (x, y, z) = (r\cos\phi, r\sin\phi, r\cot\alpha).$$



Solution: