Longest shot from the top of a hill

A gun is mounted on a hill of height $h$ above a level plain. Assuming that the muzzle speed is $v_0$ and that the path of the projectile is parabolic, show that the angle of elevation $\alpha$ for greatest horizontal range depends on $h$ and $v_0$ as follows:

$$\frac{1}{\sin^2\alpha} = 2 \left( 1 + \frac{gh}{v_0^2} \right).$$

Solution: