Reel of thread II: dynamics

A reel of thread whose spindle and rim have radii $a$ and $b$, respectively, rests on a horizontal table. The weight of the reel is $mg$ and the moment of inertia for rotations about its axis is $I$. The loose end of the thread passes under the spindle and leads off at an angle $\alpha$ above the horizontal as shown. The static frictional force between the reel and the table during rolling motion is $f \leq \mu_s N$, where $N$ is the normal force and $\mu_s$ is a constant. Consider the range $0 \leq \alpha \leq \pi$ of angles.

(a) For a given tension not too strong to make the reel roll without slipping, find the angular acceleration $\dot{\omega}$, the frictional force $f$, and the normal force $N$.

(b) For the three cases $\alpha = 0, \pi/2, \pi$ find the direction (clockwise or counterclockwise) of the angular acceleration $\dot{\omega}$ and the direction (left or right) of the frictional force $f$.

(c) For the three cases $\alpha = 0, \pi/2, \pi$ find the maximum possible value of $|\dot{\omega}|$ for rolling without slipping.

Solution: