## [mex150] Harmonic oscillator with friction

The equation of motion of a harmonic oscillator with Coulomb damping (friction) has the form

$$\ddot{x} + \alpha \operatorname{sgn}(\dot{x}) + \omega_0^2 x = 0,$$

where $\omega_0^2 = k/m$ is the angular frequency of the undamped oscillator and $\operatorname{sgn}(\dot{x})$ denotes the sign $(\pm)$ of the instantaneous velocity.
(a) Show that the solution for $n\pi \leq \omega_0 t \leq (n+1)\pi, n = 0, 1, 2, \ldots$ and $x(0) = A_0 + \beta, \dot{x}(0) = 0$ has the form $x(t) = A_n \cos(\omega_0 t) + (-1)^n \beta$. Find the constant $\beta$, the maximum value of $n$, and the amplitudes $A_n$.
(b) For the case $\alpha = 1\text{cm/s}^2$, $\omega_0 = 1\text{rad/s}$, and $x(0) = 9\text{cm}$, find the time it takes the system to come to a halt and the total distance traveled. Plot the phase portrait in the $(x, \dot{x}/\omega_0)$-plane for this particular case.

**Solution:**