

[mex156] Spherical pendulum: reduction to quadrature

A particle of mass m in a uniform gravitational field g is constrained to move on the surface of a sphere of radius ℓ .

- (a) Find the Lagrangian $L(\theta, \phi, \dot{\theta}, \dot{\phi})$, where the range of the polar angle is $0 \leq \theta \leq \pi$ and the range of the azimuthal angle is $0 \leq \phi \leq 2\pi$.
- (b) Derive the two Lagrange equations.
- (c) Identify two conservation laws.
- (d) Reduce the general solution for $\theta(t), \phi(t)$ to quadrature.

Solution: