[mex166] Precession of the perihelion: orbital differential equation

Consider the Kepler problem with a correction term reflecting relativistic effects:

$$V(r) = -\frac{\kappa}{r} - \frac{\gamma}{r^3}, \qquad \kappa = GmM, \quad \gamma = \frac{G\ell^2 M}{c^2 m}.$$

(a) Show that the resulting orbital differential equation acquires a nonlinear term as follows:

$$\frac{d^2u}{d\vartheta^2} + u = \frac{1}{p} + \alpha u^2, \qquad \frac{1}{p} = \frac{m\kappa}{\ell^2}, \quad \alpha = \frac{3GM}{c^2}.$$

(b) Calculate the angle $\delta\vartheta$ of precession per cycle of the perihelion by a perturbative solution.

Solution: