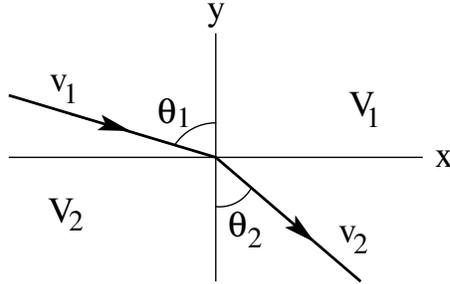


[mex167] Mechanical refraction

A particle of mass m moving in the xy -plane is subject to a potential energy which assumes the constant value V_1 at $y \geq 0$ and the constant value V_2 at $y < 0$. Let us assume that $V_2 < V_1$. Use conservation laws to show that if the particle approaches the x -axis with speed v_1 at an angle θ_1 as shown, it will proceed with a different speed v_2 at a different angle θ_2 after crossing the line where the potential energy changes abruptly. Show in particular that the relation

$$n \equiv \frac{\sin \theta_1}{\sin \theta_2} = \sqrt{1 + \frac{2}{mv_1^2}(V_1 - V_2)},$$

between the two angles holds, where n plays the role of index of refraction.



Solution: