[mex167] Mechanical refraction

A particle of mass $m$ moving in the $xy$-plane is subject to a potential energy which assumes the constant value $V_1$ at $y \geq 0$ and the constant value $V_2$ at $y < 0$. Let us assume that $V_2 < V_1$. Use conversation laws to show that if the particle approaches the $x$-axis with speed $v_1$ at an angle $\theta_1$ as shown, it will proceed with a different speed $v_2$ at a different angle $\theta_2$ after crossing the line where the potential energy changes abruptly. Show in particular that the relation

$$n \equiv \frac{\sin \theta_1}{\sin \theta_2} = \sqrt{1 + \frac{2}{mv_1^2}(V_1 - V_2)},$$

between the two angles holds, where $n$ plays the role of index of refraction.

Solution: