Scattering from a spherical potential well

Consider a spherical potential well of depth $U$ and radius $a$, described by the potential energy $V(r) = -U \Theta(a - r)$. According to [mex167], the path of an incident particle with energy $E$ encountering this potential will then be that of a ray of light refracted from a sphere with refractive index $n = \sqrt{1 + U/E}$. (a) Calculate the maximum scattering angle as a function of $n$. (b) Show that the scattering cross section has the form

$$
\sigma(\theta) = \frac{a^2 n^2}{4 \cos(\theta/2)} \frac{[n \cos(\theta/2) - 1][n - \cos(\theta/2)]}{[n^2 + 1 - 2n \cos(\theta/2)]^2}.
$$

Solution: