[mex169] Elliptic and hyperbolic orbits

Calculate the orbital integral (a) for an orbit with energy $E > 0$ and angular momentum $\ell$ of the attractive central-force potential $V(r) = \frac{1}{2}kr^2$ and (b) for an orbit with energy $E < 0$ and angular momentum $\ell$ of the repulsive central-force potential $V(r) = -\frac{1}{2}kr^2$. Show that the solutions (a) and (b) can be cast into the form $\frac{x^2}{a^2} \pm \frac{y^2}{b^2} = 1$, respectively, if the Cartesian axes are suitably oriented. Find the parameters $a$ and $b$ in each case as functions of $E, \ell, m, k$. Express $E$ and $\ell$ in terms of $a, b, m, k$ in each case.

Solution: