[mex171] Lagrange equations in rotating frame

From [mex79] we know that the Lagrange equations are invariant under a point transformation. Here we use this property to transform the equation of motion of a particle in a potential $V(\mathbf{r})$ from an inertial frame to a frame rotating with constant angular velocity $\vec{\omega}$.
(a) Express the Lagrangian $\tilde{L}(\mathbf{r}, \tilde{\mathbf{v}}) = L(\mathbf{r}, \mathbf{v}) = \frac{1}{2}m\mathbf{v}^2 - V(\mathbf{r})$ in terms of the rotating-frame coordinates.
(b) Derive the Lagrange equations $(d/dt)(\partial\tilde{L}/\partial\dot{\tilde{x}}_i) - (\partial\tilde{L}/\partial\tilde{x}_i) = 0,\ i = 1, 2, 3.$
(c) Bring the resulting Lagrange equations into the form

$$m\tilde{\mathbf{a}} = -\frac{\partial V}{\partial \mathbf{r}} - 2m\vec{\omega} \times \tilde{\mathbf{v}} - m\vec{\omega} \times (\vec{\omega} \times \mathbf{r}).$$

Solution: