[mex179] Rolling inhomogeneous disk

Consider a disk of mass m and radius R composed of two homogeneous halves connected along a diameter. One half has twice the density of the other half.
(a) Find the distance b between the center of mass and the geometric center of the disk.
(b) Find the moment of inertia I_{cm} for rotations about the center of mass.
(c) Find the Lagrangian $L(\phi, \dot{\phi})$ for the rolling motion of the disk on a flat surface. Use $\phi = 0$ for the stable equilibrium position.
(d) Consider the disk being pulled by a horizontal force at constant speed across the surface. What is the maximum speed v_{max} at which the disk can roll without jumping of the ground?

Solution: