

[mex180] Driven harmonic oscillator: steady state solution

Consider the driven harmonic oscillator, $m\ddot{x} = -kx - \gamma\dot{x} + F_0 \cos \omega t$. Show that the steady-state solution has the form

$$x(t) = D \cos(\omega t - \delta), \quad D = \frac{A}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\omega^2\beta^2}}, \quad \delta(\omega) = \arctan \frac{2\omega\beta}{\omega_0^2 - \omega^2},$$

where we have used the parameters $\beta \doteq \gamma/2m$, $\omega_0 \doteq \sqrt{k/m}$, $A \doteq F_0/m$.

Solution: