Driven harmonic oscillator: kinetic and potential energy

Consider the driven harmonic oscillator, \( m\ddot{x} = -kx - \gamma \dot{x} + F_0 \cos \omega t \), in a steady-state motion. (a) Calculate the average kinetic energy \( \langle T(\omega) \rangle \), the average potential energy \( \langle V(\omega) \rangle \), and the average total energy \( \langle E(\omega) \rangle = \langle T(\omega) \rangle + \langle V(\omega) \rangle \). Use the parameters \( \beta = \gamma / 2m \), \( \omega_0 = \sqrt{k/m} \), \( A = F_0 / m \). (b) Each quantity assumes its maximum value at a different resonant frequency: \( \omega_T, \omega_V, \omega_E \). Determine each resonant frequency.

Solution: