[mex183] Quality factor of damped harmonic oscillator

(a) Consider the driven harmonic oscillator, $m\ddot{x} = -kx - \gamma\dot{x} + F_0 \cos\omega t$, in a steady-state motion. Use the parameters $\beta \doteq \gamma/2m,\ \omega_0 \doteq \sqrt{k/m},\ A \doteq F_0/m$. In [mex182] we have calculated the maximum (averaged) power input, $P_{max} = \langle P(\omega_P)\rangle$, and in [mex181] we have calculated the average energy $\langle E(\omega)\rangle$ stored in the oscillator. Determine the quality factor of the driven oscillator defined as $Q = 2\pi\langle E(\omega_P)\rangle/\langle P(\omega_P)\rangle\tau$ with $\tau = 2\pi/\omega_P$. Show that to leading or der in β/ω_0 the quality factor is equal to the amplitude ratio at resonance and at zero frequency: $Q = D(\omega_R)/D(0)$.
(b) Consider the harmonic oscillator, $m\ddot{x} = -kx - \gamma\dot{x}$, with weak damping ($\beta/\omega_0 \ll 1$) and no driving force. Determine the quality factor Q of the damped oscillator defined as 2π times the ratio of the instantaneous energy stored, $E(t)$, and the energy loss per period, $\tau|dE/dt|$. Evaluate the result to leading order in β/ω_0

Solution: