

[mex189] Can you find the Hamiltonian of this system?

Consider the Lagrangian system

$$L(q_1, q_2, \dot{q}_1, \dot{q}_2) = \frac{1}{2}m(\dot{q}_1 + \dot{q}_2)^2 - \frac{1}{2}k(q_1^2 + q_2^2).$$

(a) Find the most general solution $q_1(t), q_2(t)$ of the associated Lagrange equations. (b) Find the Hamiltonian $H(q_1, q_2, p_1, p_2)$ such that the associated canonical equations have the same solution $q_1(t), q_2(t)$. (c) Find the most general solution of $H(q_1, q_2, p_1, p_2)$.

Solution: