

[mex190] Charged particle in a uniform magnetic field

Consider a particle with mass m and electric charge q moving in a magnetic field $\mathbf{B} = B\hat{\mathbf{e}}_z$. (a) Find the Lagrangian $L(x, y, z, \dot{x}, \dot{y}, \dot{z})$ and derive the Lagrange equations from it. (b) Find the Hamiltonian $H(x, y, z, p_x, p_y, p_z)$ and derive the canonical equations from it. (c) Show that both sets of equations of motion can be brought into the form $\ddot{x} - \omega y = 0$, $\ddot{y} + \omega x = 0$, $\ddot{z} = 0$, where $\omega = qB/mc$ is the cyclotron frequency.

Solution: